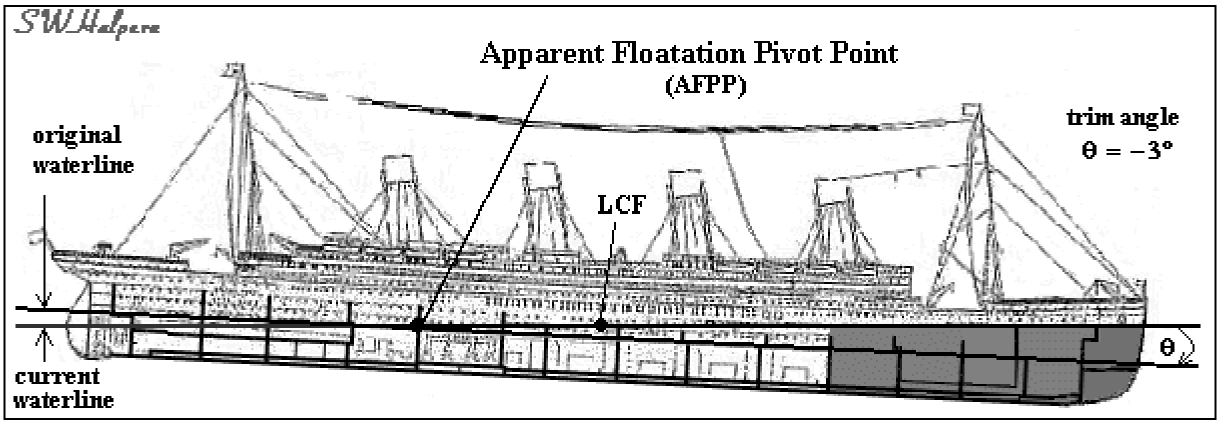
Finding the Apparent Floatation Pivot Point (AFPP)

By Samuel Halpern

To someone looking from off the side as *Titanic* slowly trimmed down by the head over a period of some 2 ½ hours following the collision with an iceberg, the ship would have appeared to be slowly pivoting about an axis on her original waterline located somewhere about 1/3 the vessel's length from the stern. This apparent axis I am calling the Apparent Floatation Pivot Point (AFPP) of the ship.



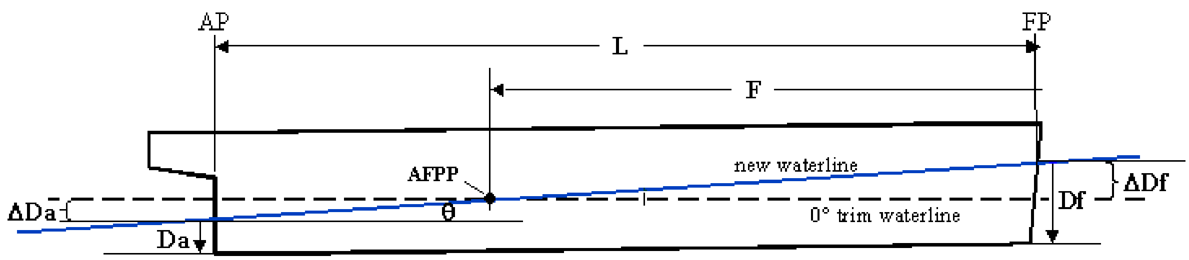
The AFPP should not be confused with what is called the ship's Longitudinal Center of Floatation (LCF), which is also a point on the waterline located at the geometric center of the current waterplane. For a ship of typical hullform, the LCF would be located not too far from the ship's midsection. If the ship is longitudinally stable at some angle of trim, any shift in weight fore or aft would cause the ship to pivot at the LCF to produce a new angle of trim. But if the ship's weight is increasing as well due to flooding, then the LCF would rise relative to the old waterplane line as the ship sank lower to establish a new waterplane as its displacement in the flooded condition increased.   However, if the floodwater is confined to several compartments, and for relatively small angles of trim (<10°), the ship will appear to rotate about the AFPP which is located on the original waterline of the vessel.

We will first show where this AFPP is located if the vessel's draft aft and draft forward are know. We will then find this point using a given flooded condition example taken from the landmark work of Hackett & Bedford. Then we show why the location of the AFPP remains about the same location as more water enters the ship in confined space, and we will also show that the angle of trim will tend to increase in approximate direct proportion to the volume of floodwater that comes into the vessel in that confined space.

The technical details follow.

Finding the AFPP When Draft Forward and Draft Aft are Known

Refer to the diagram below.



FP = Forward Perpendicular
AP = Aft Perpendicular
Df = Draft forward at given condition
Da = Draft aft at given condition
L = Length between perpendiculars
AFPP = Apparent Floatation Pivot Point
F = distance of the AFPP from the FP
θ = trim angle (taken as negative for a vessel trimmed down by the head)
ΔDf = change in draft forward from initial draft forward
ΔDa = change in draft aft from initial draft aft

The location of the **AFPP** is defined here as the intersection of the waterline for a given trim condition to the waterline for 0° trim. Again, this must not be confused with the longitudinal center of floatation (LCF) which is the geometrical center of the waterplane for a vessel in a given trimmed condition.

From the geometry of the diagram above, we can see that

$$\tan\theta = (\Delta Df + \Delta Da)/L = \Delta Df/F$$

therefore, we can solve for F to get:

$$F = L\,\Delta Df / (\Delta Df + \Delta Da) = L / (1 + \Delta Da / \Delta Df)$$

Knowing the ship's draft aft and draft forward for any condition, we can easily get the ship's angle of trim. It is simply:

$$\theta = \arctan[(Da - Df) / L]$$

Example

Start with Hackett & Bedford's intact condition for the ship which we will call condition C0.

They had: **L** = 850'; **Df** = 30.75'; **Da** = 33.75' just before the accident.

The intact trim was therefore θ = arctan [(**Da** - **Df**) / **L**] = arctan [(33.75 – 30.75)/850] = + 0.20° which is a very slight trim up by the head of 2/10 of a degree which would be unnoticeable.

Now take their flooded condition C3 as an example, after the ship had taken in a little over 14,000 tons of floodwater.

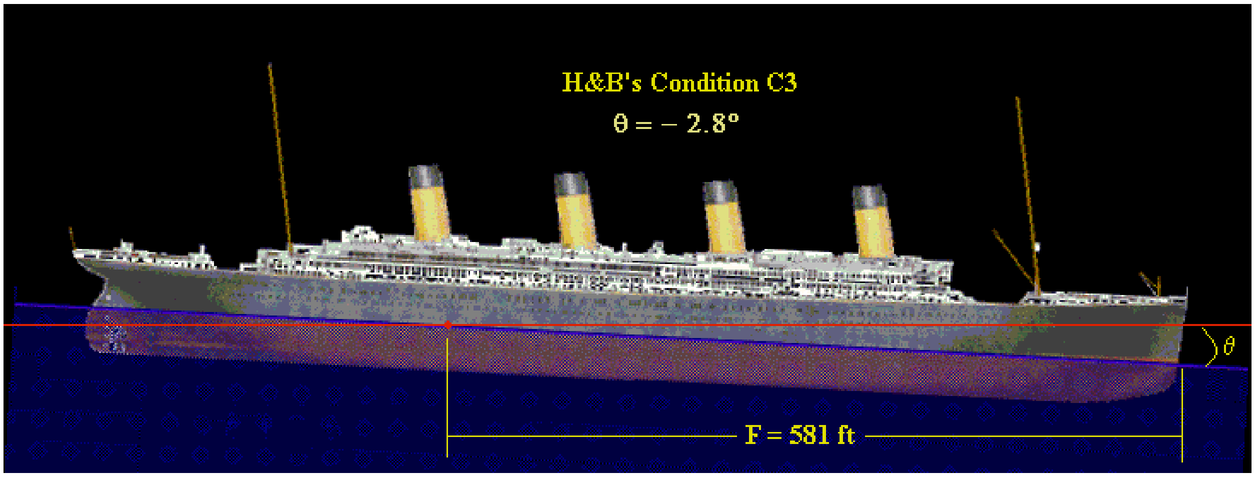
That gave them drafts of: **Df** = 61.46'; **Da** = 19.52'

Therefore, Δ**Df** = 61.46-30.75 = 30.71'; Δ**Da** = 33.75-19.52 = 14.23'

and solving for F we get, **F** = 850'/(1 + 14.23/30.71) = 580.9' aft of the FP for condition C3.

The trim angle for C3 is: θ = arctan [(**Da** - **Df**) / **L**] = arctan [(19.52 – 61.46)/850] = – 2.82° showing the ship is down by the head by almost 3 degrees.

This condition is shown in the figure below. The new waterline is shown in red, the original waterline in blue, and the AFPP, the point on the old waterline that the ship appeared to have rotated about, is marked by the red dot at the intersection of the new waterline with the original waterline.



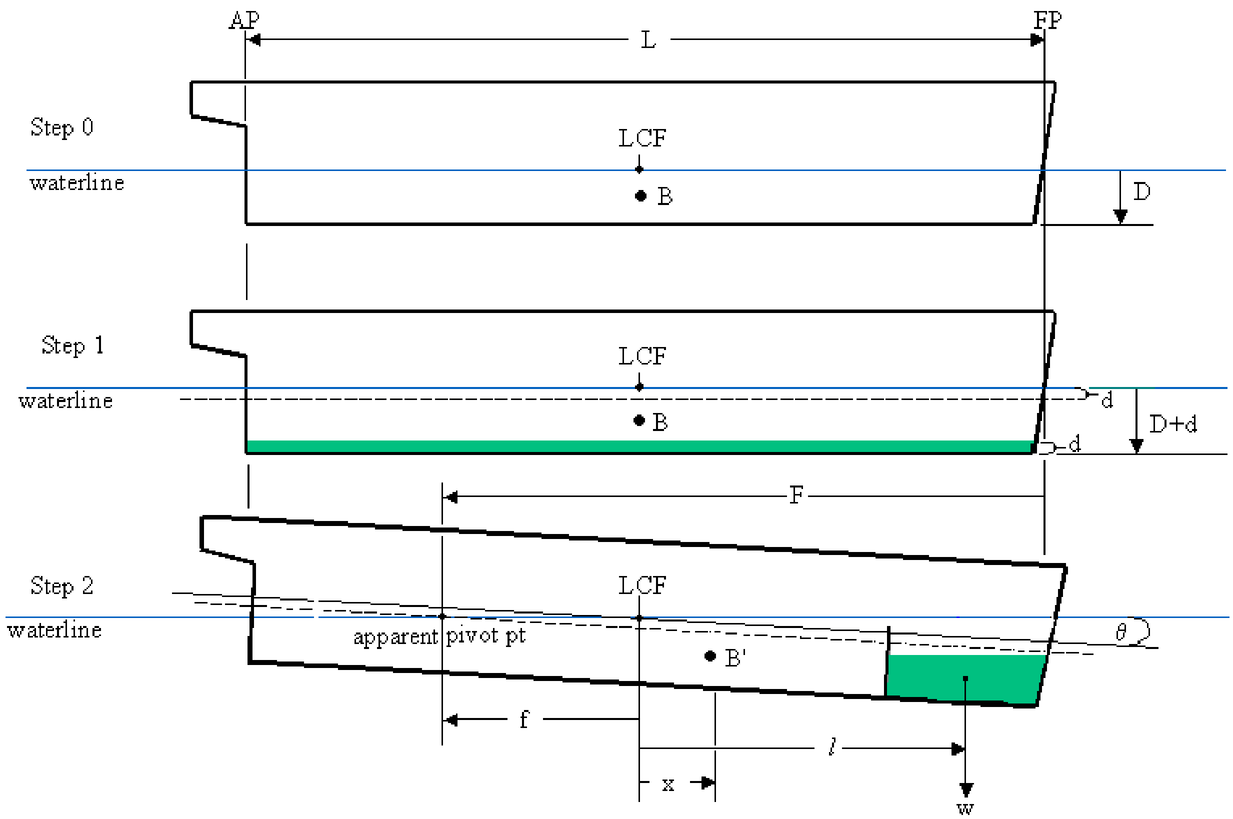
For *Titanic* at 2:15pm, when Charles Lightoller jumped into the water, the ship's crow's nest was level with the sea, and the sea was then coming over the roof of the wheelhouse. From this description we find that the ship was then about 10° down by the head. We also find from the intersection of the this waterline with ship's intact condition waterline that the AFPP was then about 585 feet aft of the FP on the ship's centerline, or just 4 feet aft of the location shown above for the vessel trimmed down by about 3°.

Why the AFPP Would Stay Approximately in the Same Place For Small Angles of Trim

Refer to the diagram below. For simplicity, let us take a vessel with an approximately symmetrical shaped hullform and approximate rectangular shaped cross sections. Under these assumptions, the vessel's Longitudinal Center of Flotation (LCF) will be located about at amidships.

The intact condition of the vessel is shown in **Step 0.**

In the diagram, **L** = length between perpendiculars, **D** = draft of vessel in the intact condition, and **B** is the center of buoyancy which is assumed to be acting amidships for this vessel in the intact condition with 0° trim. It's location is about **D/2** up from the keel. The displacement of the ship would be proportional to the area **LD** multiplied by the mean breath of the vessel, **b** (not shown).



Next for **Step 1**, let us flood the vessel so water rises to a level **d** along the entire bottom as shown. The volume of water to enter the ship is proportional to the area **Ld** as highlighted in green. Because of the added weight of this water in the bottom of the hull, the vessel will sink down **d** feet to increase its under water volume by the same amount. Thus the draft for Step 1 now becomes **D+d** as shown. The total displacement of the vessel is now proportional to the area **L(D+d)**, and the new waterline is **d** feet higher than the original waterline as shown. (Original waterline is the dashed line.) The LCF is still amidships but located on the new waterline as marked, and the center of buoyancy, B, is now **(D+d)/2** above the keel and still located amidships with the ship at 0° trim.

Now in **Step 2**, let's pump all the floodwater into a forward compartment space as shown. The floodwater volume has not changed so the total displacement of the ship is still proportional to the area **L(D+d)** as in Step 1 above. But because of the shifted weight, the ship must trim down by an angle **θ** about the LCF until the center of buoyancy is shifted to **B'** as shown. As in transverse stability, the shift in center of buoyancy is given by $\mathbf{x = L^2\ \theta\ /\ 12(D+d)}$ for small angles of trim. The sum of the moments (taken about the original CG location on the midship cross section) must still vanish for a ship in stable trim. The weight of the ship excluding the floodwater is still proportional to the original displacement area **LD** and is directed downward along the line amidships. It's moment arm is 0. The weight of the floodwater **w** is still proportional to the area **Ld** of Step 1 and directed downward. But in Step 2 it is acting at a point ***l*** feet from amidships as shown. The buoyant force of the ship in Step 2 is equal to the total displacement of the ship as in Step 1, which is proportional to the area **L(D+d)**. But in Step 2 it is acting upward at the location **B'** with a moment arm of **x** as shown**.** Thus, summing all the moments, we have:

$$\mathbf{L\ (D+d)\ x = Ld}\ \boldsymbol{l}$$

And substituting for x, we get:

$$\mathbf{L\ (D+d)\ L^2\ \theta\ /\ 12(D+d)\ = Ld}\ \boldsymbol{l}$$

Solving this for the angle of trim **θ**, we get:

$$\boldsymbol{\theta} = \mathbf{12\ d}\ \boldsymbol{l}\ \mathbf{/L^2}$$

But the area **Ld** is proportional to the floodwater volume that entered the hull. So let that floodwater volume be ***V*** **= bLd,** where **b** is the mean breadth of the ship**.** Now if we both multiply and divide the right side of the above equation for θ by **bL**, and then substitute *V* for bLd in the numerator, we get the simple relation:

$$\boldsymbol{\theta} = \mathbf{12}\ \boldsymbol{V\ l}\ \mathbf{/\ (bL^3)} \quad \text{for } \theta \le 10°$$

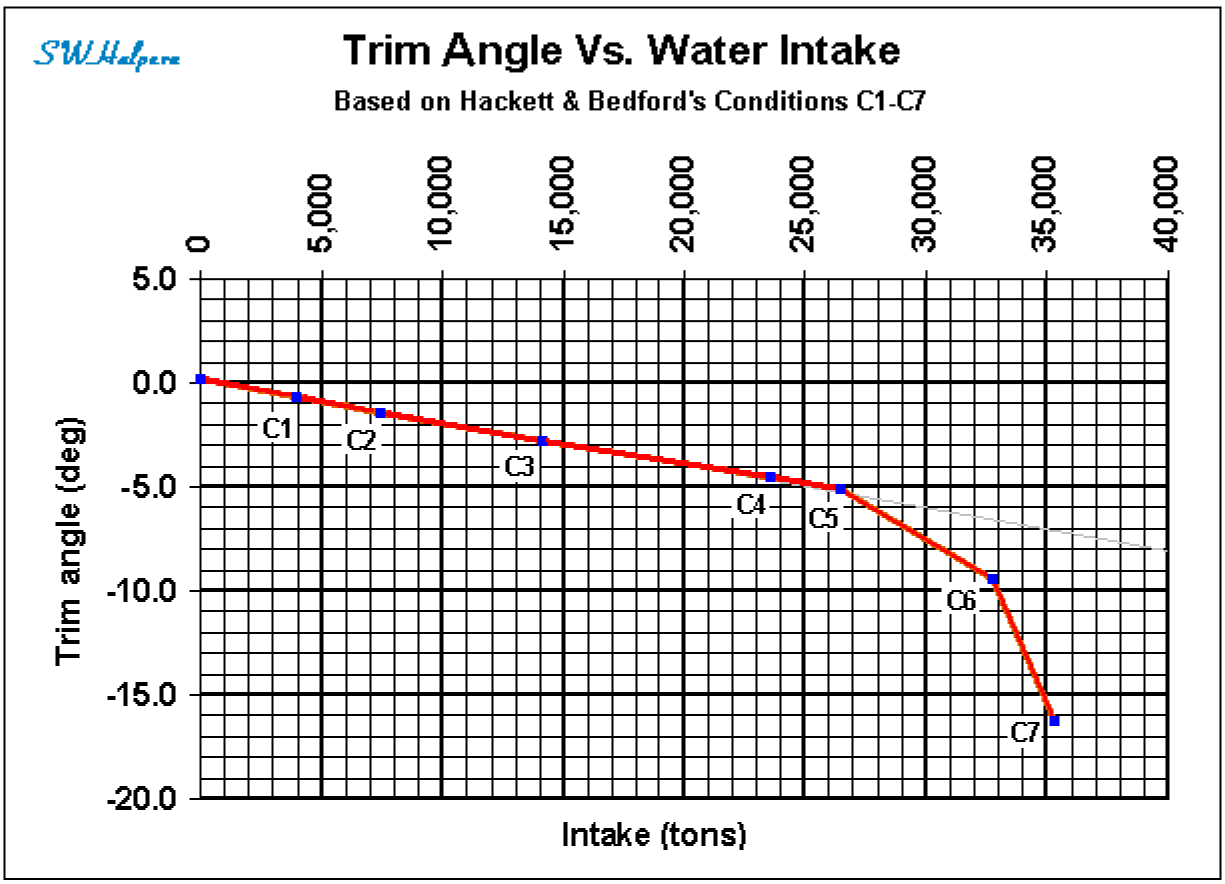
So we see that the trim angle **θ** is directly proportional the volume of floodwater that entered the hull when confined to the space the geometric center of which is ***l*** feet ahead of amidships. This result is valid for small trim angles (app. ≤ 10°).

Now consider the intersection of the new waterline shown in Step 2 with the original waterline of the intact ship (shown by the dashed line in Step 2). This point, what we have been calling the AFPP, is located **f** feet aft of amidships (or **F = f + L/2** aft of the FP). From the geometry of the situation we see that **f = d/tanθ** which for small angles of **θ** equals:

$$\mathbf{f = d/\theta\ = d/(12\ d}\ \boldsymbol{l}\ \mathbf{/L^2) = L^2/(12}\ \boldsymbol{l}\mathbf{)}$$

a result that is inversely proportional to the distance of the geometric center ***l*** of the confined floodwater space taken from the amidships section. So as long as the flooding can be confined to the compartments forward of a given bulkhead, and assuming the location of the geometric center of the floodwater volume remains about in the same cross sectional plane of the ship, the location of the apparent floatation pivot point along the original waterline of the ship remains about in the same place even as the ship goes down by the head to about 10° of trim. The ship would appear to rotate about the this pivot point as seen from off the ship's side. Notice also, that if the floodwater starts to move aft of the original confined forward space, the distance ***l*** will tend to shorten as the geometric center of floodwater volume starts to move aft. Then the distance of this pivot point will also move aft because **f** will increase as ***l*** decreases.

I plotted the trim angle **θ** as a function of water intake (in tons) for H&B's conditions C1-C7. It is shown below.



Notice that the curve is approximately linear through condition C5 as water was mostly confined to the forward 6 compartments from the very beginning. This is in good agreement with what is expected from our simplified analysis. I also found the location of the apparent floatation pivot point F where the new waterline would cross the intact condition waterline. The results are shown in the table below along with the resulting trim angles and the amount of floodwater that entered the ship for H&B's flooded conditions C1 – C7 and the intact condition C0.

H&B condition	Da (ft)	Df (ft)	Water intake (long tons)	Angle of Trim (deg)	F (ft aft of FP)
C0	33.75	30.75	0	0.20	N/A
C1	29.96	39.33	3,975	-0.63	590
C2	25.85	47.54	7,450	-1.46	578
C3	19.52	61.46	14,131	-2.82	581
C4	12.21	78.9	23,607	-4.49	587
C5	9.25	85.15	26,438	-5.10	586
C6	-11.38	129.67	32,741	-9.42	584
C7	-43.85	203.94	35,312	-16.25	587

Notice that in the above table the apparent pivot point location remains between 578 and 590 feet aft of the Forward Perpendicular, about under the location of the 4th funnel on the original intact condition waterline. This can be seen in the diagram above showing H&B's C3 flooded condition.

The above results account only for the differences in trim of the vessel as more and more floodwater entered the hull. They do not account for any changes in list due to asymmetrical flooding in the transverse direction. It should be noted that *Titanic* had assumed an early list of about 5° to starboard that later had corrected itself. Late in the sinking process she took on a list of about 10° to port. To an observer looking from afar, they would see the combined affects of a vessel trimmed down by the head as well as assuming a slight list to one side. With a slight list being carried, the location of the AFPP would still apply but only on the vessels longitudinal centerline because the outside of the hull on the side to which the ship is listing would appear to be lower while that on the opposite side would appear to be higher than a 0 list condition vessel.

Summary and Conclusions

We have shown that for floodwater confined to few compartments forward, a vessel will trim down by the head in approximate direct proportion to the volume of floodwater to enter the ship for relatively small angles of trim. We have also shown the to an observer looking toward the ship from the side, the ship would appear to rotate about a point on the original waterline (the AFPP) that would remain approximately in the same place over relatively small angles of trim. We have also shown how to determine where this point is if the vessel's draft aft and draft forward are know for the intact condition and for any given flooded condition.