A MATTER OF STABILITY AND TRIM

By Samuel Halpern

INTRODUCTION

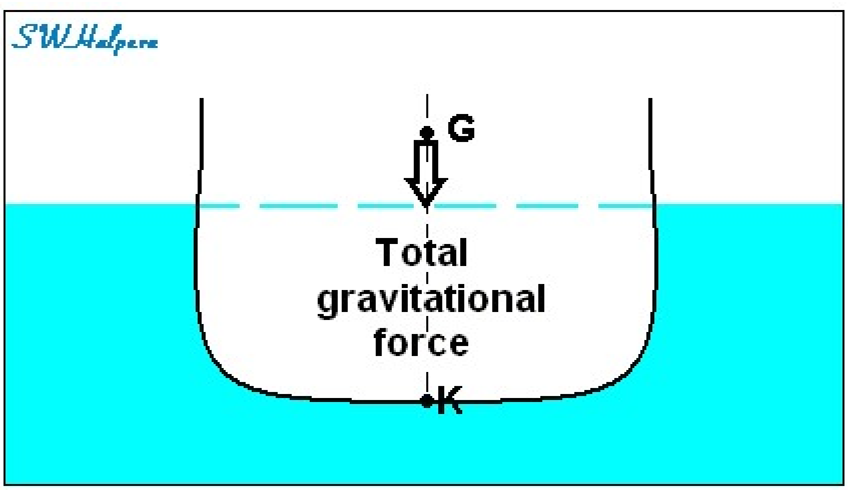
This short paper deals with the location of *Titanic's* Center of Buoyancy (B), Center of Gravity (G) and Metacenter Height (M) on the night of April 14, 1912. The location of these three points determine how stable the ship was in her intact condition before she was damaged by an encounter with an iceberg. It is also of interest to anyone wanting to build an accurate floating replica of the ship. In addition to these, the ship's longitudinal center of floatation (LCF) is also derived.

BACKGROUND

When a ship is floating at rest in calm water, it is acted upon by two sets of forces:

1. the downward force of gravity, and
2. the upward force of buoyancy.

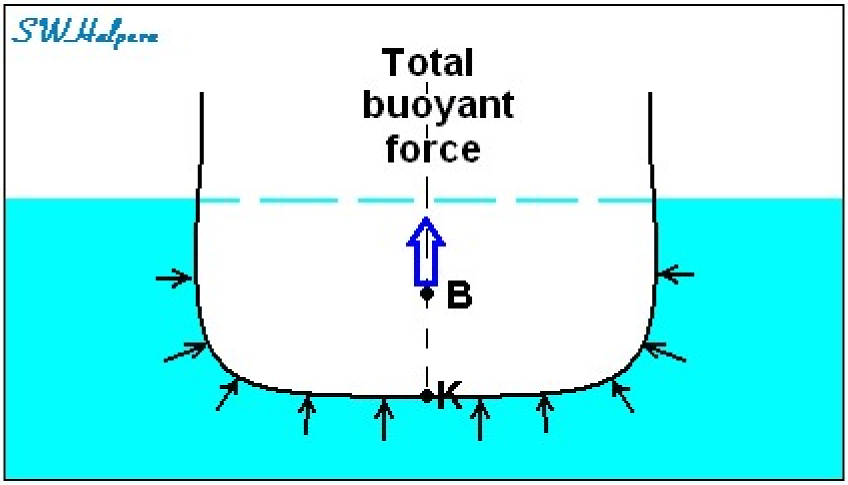
The force of gravity is the result of a combination of all downward forces including the weights of all parts of the ship's structure, all the equipment, cargo, fuel and personnel. This combined force is the weight of the ship, and may be considered as a single force which acts downward through a single point called the center of gravity (G). This force is equal to the ship's displacement in tons and usually lies on the ship's centerline near the ship's amidship section under normal trim conditions.



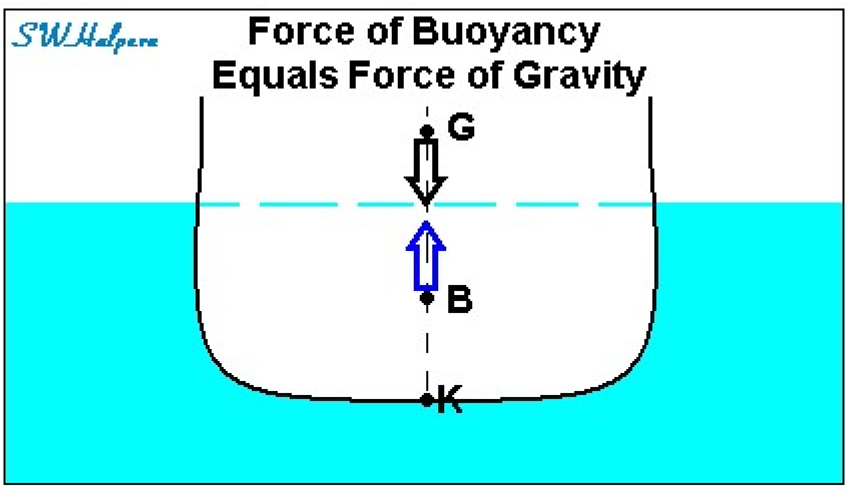
The force of buoyancy is also a combined force which results from the pressure of seawater on all parts off the ship's hull below the waterline. At almost every point on the submersed hull, the pressure of the seawater acting on the hull can be broken into two perpendicular components, one in a horizontal direction and another in a vertical direction.[1] Horizontal pressures on the hull of a ship cancel each other under normal conditions, as there are

[1] On parts of the submerged hull that are truly vertical, there is only a horizontal component of pressure acting on it; while on those parts of the submerged hull that are truly horizontal, there is only a vertical component of pressure acting on it.

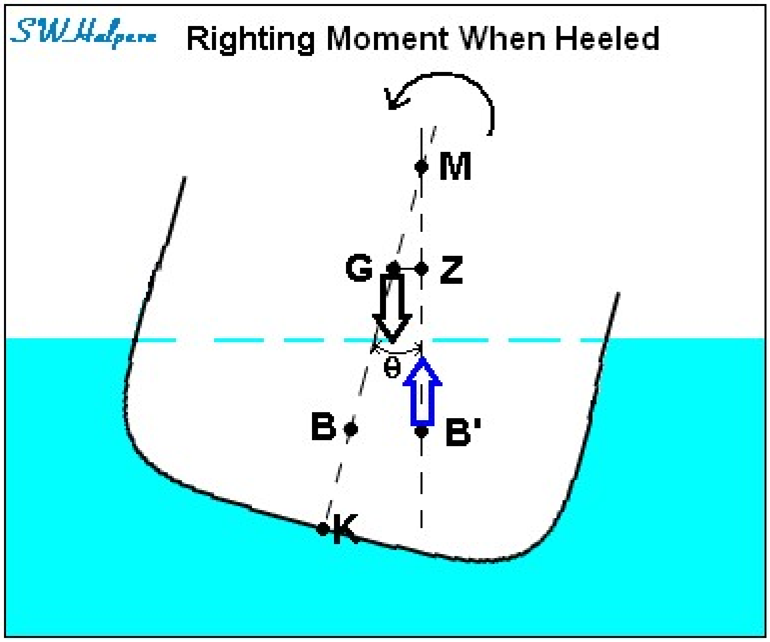
equal forces acting in opposite directions. However, the vertical pressures add up to a single force of buoyancy equal to the ship's displacement in tons that may be regarded as acting vertically upward through a single point called the center of buoyancy (B). The center of buoyancy is the geometric center of the ship's underwater body and lies on the centerline and usually near the amidship section when the ship is on an even keel. Its vertical height above the keel is usually a little more than half the draft of the vessel.



When a ship is at rest on an even keel in calm water, the forces of buoyancy (B) and gravity (G) are equal and opposite, and lie in the same vertical line, as shown in the figure below.



A ship may be disturbed from rest by conditions which tend to make it heel over to an angle. These include such things as wave and wind action, forces during a turn, shifting of weights or location of weights off-center. When a disturbing force exerts an inclining moment to the vessel, the ship's underwater body changes shape. The center of the underwater volume is shifted in the direction of the heel which causes the center of buoyancy to relocate off of the vessel's centerline (originally at B) and move to the geometric center of the new underwater body (at B'). As a result, the lines of action of the forces of buoyancy and gravity are no longer acting in the same vertical line, but are separated thereby creating what is called a righting moment that wants to restore the ship back onto an even keel as shown in the next diagram.



The righting moment is usually taken about the center of gravity point. It is the product of the force of buoyancy times the distance GZ that separates the line of action of the buoyancy force from the center of gravity as shown. The distance GZ is called the "righting arm." Since the force of buoyancy must equal the weight of the ship, the restoring moment is simply equal to the ship's displacement in tons times the length of the righting arm in feet. The result will be in foot-tons.

A ship's metacenter (M) is the intersection of two successive lines of action of the force of buoyancy as the ship heels through a very small angle. When the ship is on an even keel, the buoyant force is directed along the line from B through G. When the ship is heeling, the buoyant force is directed upward from B' passing through Z to the point where it intersects the original buoyant force line. For a small angle of heel, about 10° or less, that intersection point will remain essentially stationary. It is called the initial position of the transverse metacenter. The initial position of the metacenter is most useful in the study of translational stability because of the relationship between the "righting arm" (GZ) and the "metacentric height" (GM) above G. That relationship is given by the equation:

$$\mathbf{(GZ) = (GM) \times \sin\theta}$$

and for small angles of θ, $\mathbf{(GZ) \approx (GM) \times \theta / 57.3}$, where θ is expressed in degrees.

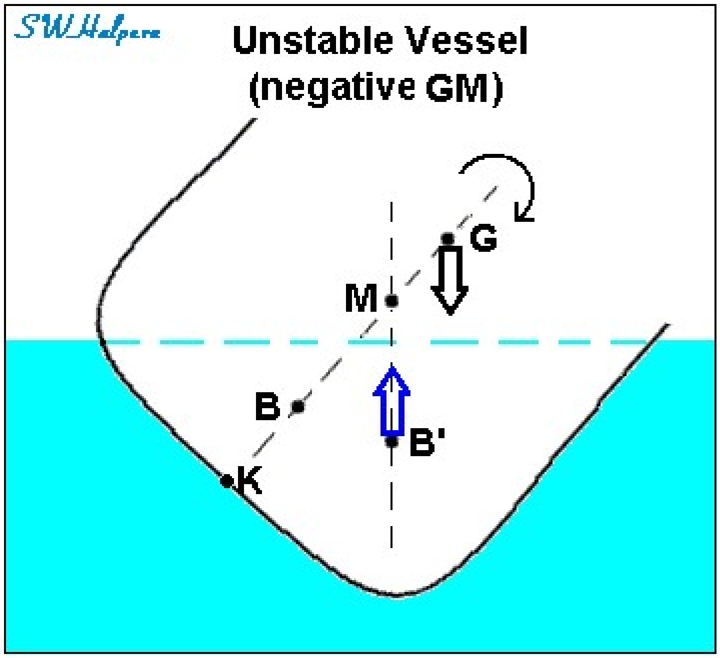
Since the righting moment is equal to the force of buoyancy times (GZ), and since the force of buoyancy must equal the ship's displacement weight (W) in tons, the righting moment in foot-tons for small angles of heel is given by:

$$\textbf{Righting Moment} \approx \mathbf{(GM) \times \theta \times W / 57.3}$$

where again (GM) is in feet, W is in tons, and θ is in degrees (for $\theta \le 10°$).

It should also be pointed out that the distance (BM), from the center of buoyancy (B) to the metacenter (M) when the ship is on even keel is called the "metacentric radius."

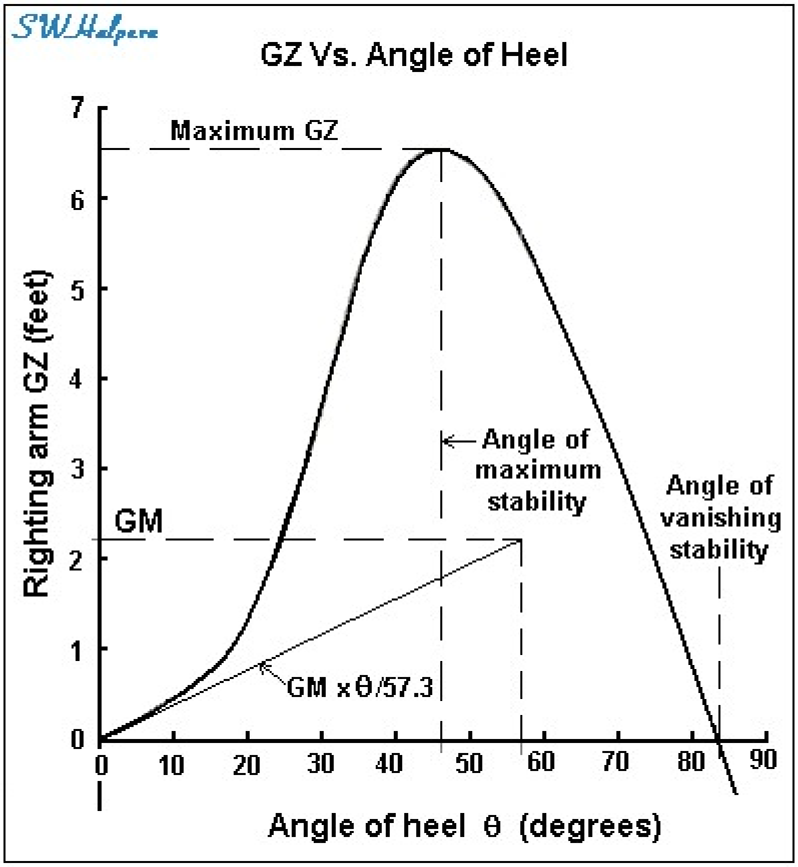
We can see from the above that (GM) is an indicator of the ship's initial stability. If M is above G, as shown in the previous figure, the metacentric height is positive and the moment which develops when the ship is inclined will be a righting moment tending to bring the vessel back to an even keel. The ship is stable. But if M falls below G, then the metacentric height is negative, and the moment that develops is an upsetting moment. In this case, the ship is unstable and will want to capsize. This is shown below.



If the metacentric height (GM) of a ship is too large, the righting arms that develop at small angles of heel will also be large. Such a ship is very "stiff" and will roll with a short period and large amplitude as it tries to follow the slope of the waves. It can become very uncomfortable for passengers and crew, especially in a moderate to heavy seaway. On the other hand, if the metacentric height of a ship is small, the righting arms that develop will also be small. Such a ship is considered "tender" and will have a long roll period. If the metacentric height is too small, the risk of the ship capsizing in rough weather increases, and it also puts the vessel at risk of developing large angles of heel if cargo or ballast should happen to shift. A ship with a very small (GM) is also less safe if the ship is damaged because it leaves less of a safety margin against capsizing.

For the majority of ship hull forms, the curve of the righting arm (GZ) as a function of angle of heel (θ) departs from an initial straight line with some increasing slope for angles beyond about 10°. As the ship heels further, its waterplane area increases and the value of (BM), the metacentric radius, also increases. This causes a greater increase in (GZ), which produces a greater righting moment as a consequence. Eventually, a point is reached where the value of (GZ) peaks. The point where that occurs is called the "angle of maximum stability" and produces the greatest righting moment acting on the ship to bring it back onto an even keel. Beyond that point, the right arm decreases and reaches zero at what is called the "angle of vanishing stability." Theoretically, it is the point beyond which the ship will capsize. In reality, capsizing will occur at a somewhat smaller angle than that.

An example curve of (GZ) as a function of heeling angle is shown in the next diagram.



DETERMINING THE LOCATION OF B, G, AND M FOR *TITANIC*

The metacentric height for *Titanic* was estimated by Chris Hackett and John G. Bedford at (GM) = 2 ft 7.5 in [2.625 ft] based on 1911 data taken from *Olympic*.[2] The draft of *Titanic* after completing about 2/3 of her voyage on the night of April 14, 1912, was estimated (by naval architect Edward Wilding and presented to the Wreck Commission) at T = 32 ft. 3 in. [32.25 ft]. At that draft, *Titanic's* displacement having completed about 2/3 of her voyage was estimated at W= 48,300 long-tons.[3] Based on what we derived above, the *initial* righting moment for *Titanic* on the night of April 14, 1912 as a function of angle θ would be approximately 2,200×θ foot-tons for small angles of θ given in degrees.

To get the height above the keel for the metacenter (KM), we need to find the height above the keel for the center of buoyancy (KB) and add to it the metacentric radius (BM). This gives (KM) = (KB) + (BM). To find the height of the center of gravity above the keel (KG), we simply subtract (GM) from the height of the metacenter above the keel (KM). This gives (KG) = (KM) – (GM).

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[2] Hackett and Bedford, "The Sinking of S.S. *Titanic* - Investigated by Modern Techniques," 1996 RINA Transactions.

[3] From the entries for *Titanic* in the original record book "Particulars of Completed Ships" in Harland & Wolff archives, the change in displacement in tons per inch immersion (TPI) was listed at 143.8. The displacement at her load draft of 34 ft. 7 in. was 52,310 tons. Wilding derived a mean draft at the time of collision at 32 ft. 3 in. That is a difference of 28 inches between the vessels load draft and her draft on April 14. The change in the vessel's displacement is therefore 28x143.8 = 4,026 tons. Subtracting this number from the load displacement gives 52,310 - 4,026 = 48,284 tons for the night of Apr 14. Wilding rounded this to 48,300 for his working estimate.

The midship cross section of *Titanic* was very close to being rectangular in shape. Her Midship Section Coefficient (Cm), a measure of how close the underwater midship section of the ship is to a rectangle, can be obtained from the ratio of the ship's block coefficient to her prismatic coefficient. The Block Coefficient (Cb) is the ratio of the immersed volume of a vessel at a certain load condition to the product of its immersed draft, length, and beam at that condition. The Prismatic Coefficient (Cp) is the ratio of the immersed volume to the product obtained by multiplying its length on the waterline by the immersed area of the midship transverse section at the load condition. Both these numbers are available from Harland & Wolff archives.[4] For *Titanic* [Hull 401], Cb=0.684, and Cp=0.705. From this we get a Midship Section Coefficient of Cm=Cb/Cp=0.970 which tells us that the area of the underwater midship section was very close to a rectangular cross section. If the entire underwater body were rectangular, then it can be shown (see Appendix A) that the metacentric radius (BM) would be given by:

$$(\mathrm{BM}) = B^2/(12\times T)$$

where T is the draft of the vessel, and B is her breadth.

However, *Titanic's* cross section was close to rectangular in the middle third of the vessel only. Thus we have to use a more exact method of finding (BM) by using the actual plan diagrams. This is described in Appendix B along with a few other parameters which are discussed below.

For *Titanic* at a draft of T=32.25 ft, the derived metacentric radius [see Appendix B] comes out to **BM = 20.86 ft**.

The height of the center of buoyancy above the keel (KB) for a hull form like *Titanic* can be accurately estimated using Morrish's formula,[5] which is:

$$\mathbf{(KB) = 1/3 \times (5\times \mathit{T}/2 - V/Awp)}$$

Where in the above T is the draft of the vessel, V is the underwater volume of the vessel at that draft, and Awp is the area of the waterplane of the vessel at that draft. Since the displacement on the night of April 14 was W=48,300 tons, the underwater volume is V=35×W=1,690,500 cu ft.[6]

The area of the water plane can be obtained by using Simpson's rule (see Appendix B) and a set of body plan half-section diagrams from Harland & Wolff. For a waterline at T = 32.25 ft above the keel, the waterplane area came out at **Awp = 59,899 sq ft**. Then applying that to Morrish's formula gives us a location for the center of buoyancy above the keel at (**KB) = 17.47 ft** (which is 54% of the ship's draft above the keel) for the night of April 14, 1912. We now have all that we need to also get the heights of the center of gravity and the metacenter above the keel.

The height of the metacenter above the keel is just **(KM) = (KB) + (BM) = 38.32 ft;** and the height of the center of gravity above the keel is simply, (**KG) = (KM) – (GM) = 35.70 ft.**

As a point of reference, F deck on *Titanic* amidships was 36 ft. 9 in. above the keel. Thus, we see that *Titanic's* center of gravity in the intact condition was about a foot below the level of F deck near amidships. Her initial transverse metacenter would have been 2 feet 7.5 inches above that point. All this can be seen in the diagram below for *Titanic's* amidship cross-section.

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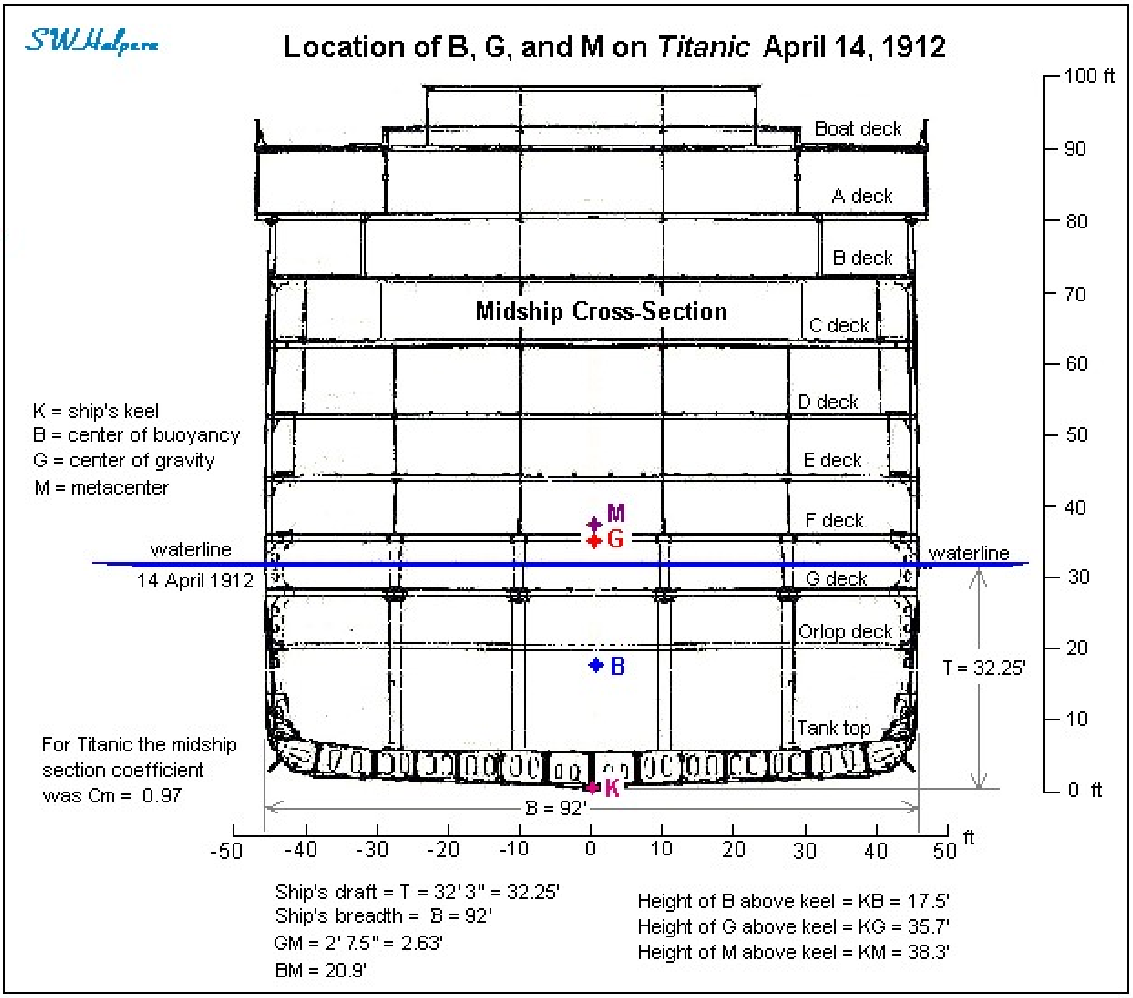
[4] From entries for *Olympic* and *Titanic* taken from "Particulars of Completed Ships," Harland & Wolff archives.
[5] Edward L. Attwood, *Theoretical Naval Architecture*, Longmans, Green and Co., 1922.
[6] The density of seawater is taken at 35 cu ft/ton. All tons used are British long tons, 2240 pounds.



It should also be mentioned that Harland & Wolf naval architects had once calculated the location of the center of buoyancy in the fore-aft direction for *Olympic* assuming a displacement of 51,290 tons at a draft of 34 ft. 3 in. for the ship on an even keel. They found the center of buoyancy to be located 5.7 feet *aft* of amidships. The center of gravity, of course, would be directly above the center of buoyancy in that condition. In doing this they used a set of Bonjean curves.[7]

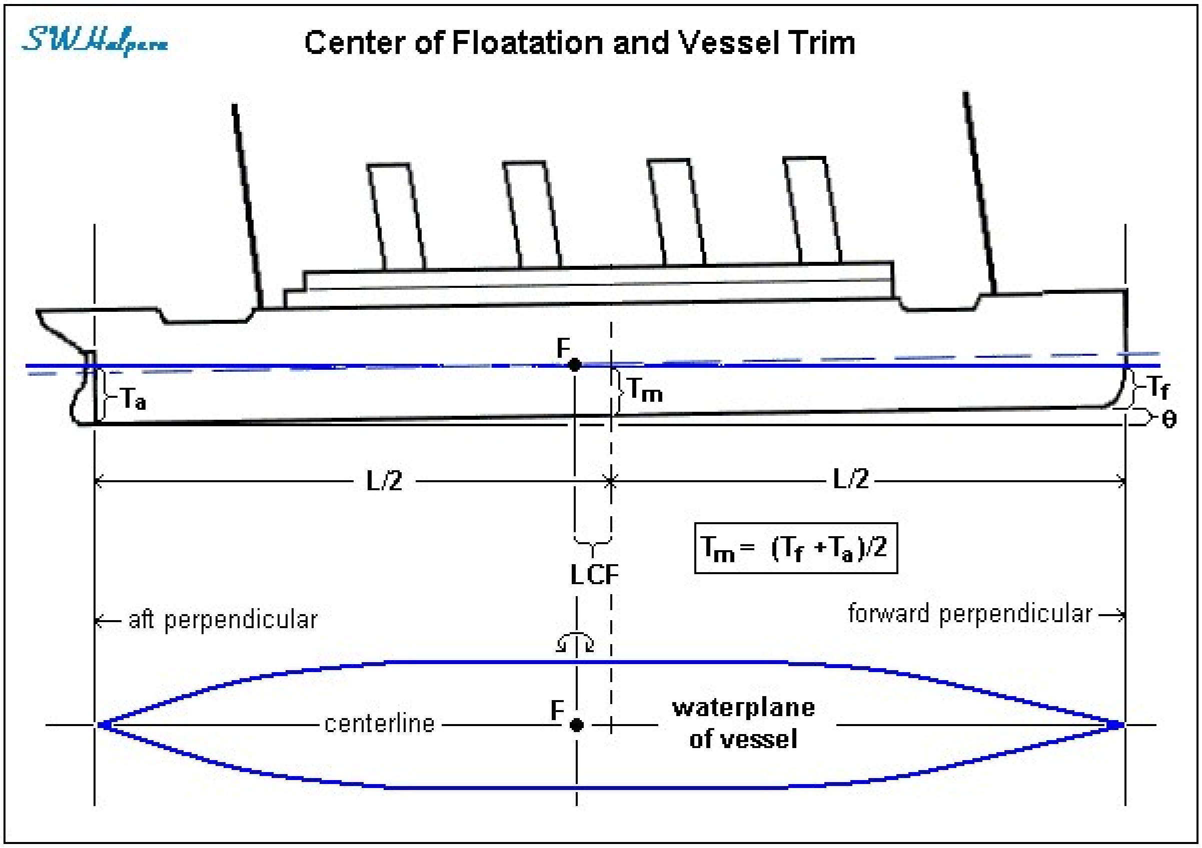
DETERMINING *TITANIC'S* LONGITUDINAL CENTER OF FLOATATION

The center of floatation (F) of a vessel is the geometric center of the vessel's waterplane. It is the point about which the vessel will list and trim.[8] When the ship is upright and carrying no

[7] Bonjean curves are curves of immersed cross-sectional area plotted against the vessel's draft and are drawn on the profile of the ship at each station. The use of Bonjean curves allows the underwater volume to be found for waterlines that are not necessarily parallel to the vessel's baseline such as when the ship is not floating on an even keel. Waterlines at any angle to the vessel's baseline can be superimposed upon the profile, and it becomes possible to read off the immersed areas by drawing lines parallel with the baseline from the intercept of the waterline with the section to the Bonjean curve for that section. In calculation the underwater volume and the location of the center of buoyancy for *Olympic*, H&W architects used a set of 21 equally spaced stations.

[8] When the ship lists to port or starboard, or trims down by the bow or stern, or changes draft, the shape of the waterplane will change. Thus, the location of the geometric center of the waterplane will move and lead to a change in the location of the center of flotation.

list, the center of floatation in the transverse direction will be on the vessel's centerline. The location of the center of floatation in the fore-and-aft direction, called the longitudinal center of floatation (LCF), is the point about which the vessel inclines or trims in the fore-and-aft direction. This location is important in longitudinal stability, and is specified as forward or aft of amidships, or as a length forward of the aft perpendicular or aft of the forward perpendicular. The location of F is shown in the diagram below for a vessel slightly trimmed by the stern such as *Titanic* on the night of April 14, 1912.



In the above diagram, ***Ta*** is the draft of the vessel aft, ***Tf*** is the draft of the vessel forward, and ***Tm*** is the mean draft of the vessel which is located at the amidships point, halfway between the perpendiculars where the drafts are taken. For *Titanic* on the night of April 14, 1912, the draft aft was Ta = 33 ft. 9 in., the draft forward was Tf = 30 ft. 9 in., and her mean draft Tm = 32 ft. 3 in. In this condition, the ship was trimmed down by the stern, $Ta - Tf$ = 3 ft. 0 in.

The angle of trim, θ, is given by:

$$\theta = \arctan (Ta - Tf)/L$$

which for *Titanic* on the night of April 14, comes out to **θ = 0.20°**.

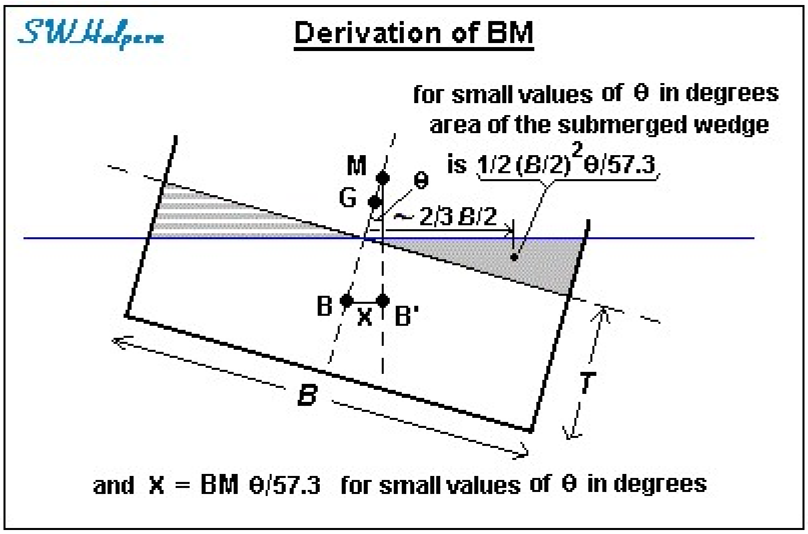
The derivation of the longitudinal center of floatation (LCF) for *Titanic* is explained in Appendix B. For *Titanic* in the intact condition, **LCF = 10.0 ft aft of amidships** on the centerline of the vessel 32.25 ft. above the keel.

ACKNOWLEDGEMENT

I would like to thanks Captain Charles Weeks of the Maine Maritime Academy for his many helpful suggestions and encouragement.

APPENDIX A – Deriving the Metacentric Radius (BM) for a Rectangular Cross Section

The metacentric radius (BM) can easily be derived for a ship with rectangular cross section. Consider the rectangular cross section of a vessel with an angle of heel θ as shown in the diagram below.



The center of area of a narrow submerged wedge can be shown to be located about 2/3 the distance of half the ship's beam width from the vessel's centerline.[9] This length is the moment arm for the submerged wedge area. In summing the moments in calculating the shift in the center of buoyancy from B to B' we have to take twice the submerged wedge area because the right side gains submerged buoyancy area while the left side looses the same amount of submerged buoyancy area. To get the shift in the center of buoyancy we therefore simply multiply twice the area of the submerged wedge by its distance from the centerline ($2/3\times B/2$), and divide that by the total underwater area which is equal to the vessel's beam multiplied by its draft ($B\times T$).[10] Since the area of the wedge for small angles of heel is $\frac{1}{2}\times(B/2)^2\times\theta/57.3$, we obtain:

$$\mathbf{X} = 2\times[\tfrac{1}{2}\times(B/2)^2\times\theta/57.3]\times 2/3\times B/2\ /(B\times T) = B^2\times(\theta/57.3)/(12\times T)$$

Now from the diagram above, we also see that the distance X, between B and B', must equal (BM)×sin θ, which for small angles of θ, is nothing more than **X = (BM)×θ/57.3** when θ is given in degrees. Therefore, equating the two terms for X we get,

$$(\mathrm{BM})\times\theta/57.3 = B^2\times(\theta/57.3)/(12\times T)$$

[9] The centroid of a triangle is the intersection of its medians and can be found by averaging the coordinates of its three vertices.

[10] It should be obvious from the diagram of the cross section for the heeled vessel that the total underwater area remains the same in the heeled condition since the vessel looses as much area as it gained when it heeled over. It is only the location of the center of buoyancy that is shifted.

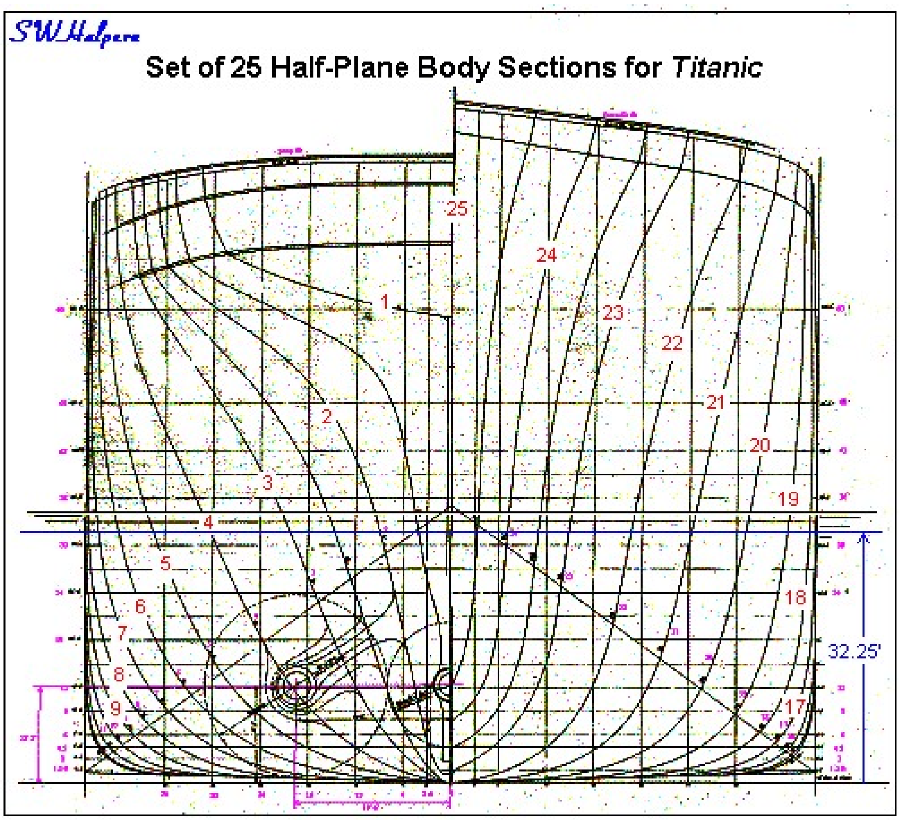
which simplifies to,

$$(\mathbf{BM}) = \boldsymbol{B}^2/(12\times\boldsymbol{T})$$

the result we were looking for.

APPENDIX B – Deriving *Titanic's* Waterplane Area (Awp), Longitudinal Center of Floatation (LCF), and Metacentric Radius (BM)

The waterplane area for *Titanic* can be obtained from a set of Harland & Wolff Body Plans for *Olympic* and *Titanic* like those shown below.



Using N=25 equally spaced half-plane sections, we can apply Simpson's first rule for integration to determine the waterplane area at a height of 32 ft 3 in above the keel. The rule formula using half-plane sections is:

$$\mathbf{Awp = 2 \times 1/3 \times S\ (Y_1 + 4Y_2 + 2Y_3 + 4Y_4 + 2Y_5 + 4Y_6 + \ldots + 2Y_{23} + 4Y_{24} + Y_{25})}$$

where Y_n is the distance from the ship's centerline to the outside shell for the n^{th} section at the waterline height chosen. The value of S is the spacing between sections, which is equal to the length of the waterline, L, divided by N-1. In our case we use L=850 ft., and S=L/24=35.42 ft.[11]

[11] Station No. 25, the forward perpendicular, in the H&W Sheer and Half-Breadth Plans for *Olympic* and *Titanic* intersects the ship's stem at the level of E deck, about 58 ft. above the keel. Station No. 1, the aft perpendicular, is at the forward side of the rudder post. The length between these perpendiculars is 850 ft. The rake of the stem is 1 ft.

When applying the distances Y_n taken from the section curves to the formula above, we find that the waterplane area comes out to **Awp = 59,889 sq ft**.

Using the Simpson's rule for integration we can also find the longitudinal center of floatation (LCF) for the ship, and the metacenric radius (BM). For the LCF taken forward of the aft perpendicular, the formula is:

$$\mathbf{LCF = 2 \times 1/3 \times S^2 (0\times Y_1 + 1\times 4Y_2 + 2\times 2Y_3 + 3\times 4Y_4 + \ldots + 22\times 2Y_{23} + 23\times 4Y_{24} + 24\times Y_{25})/Awp}$$

For *Titani*c in the intact condition on the night of April 14, 1912, the longitudinal center of floatation comes out at **LCF=415.0 ft ahead of the aft perpendicular**, or **10.0 ft aft of amidships** on the waterplane.

The transverse metacentric radius (BM) is also obtained using the Simpson's rule for integration. In this case, the formula for BM becomes:

$$\mathbf{BM = 2/9 \times S \times (Y_1^3 + 4Y_2^3 + 2Y_3^3 + 4Y_4^3 + 2Y_5^3 + 4Y_6^3 + \ldots + 2Y_{23}^3 + 4Y_{24}^3 + Y_{25}^3) / V}$$

where V is the underwater volume at the draft condition, which for a displacement of W=48,300 tons, is V=35×W=1,690,500 cu ft. For *Titanic*, the metacentric radius comes out to (**BM)=20.86 ft.**

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for every 12 ft. The design load waterline, 34 ft 7 in., shows up about 2 ft. aft of the forward perpendicular, and the 32 ft. 3 in. waterline is about 2 ft. 2 in. aft of the forward perpendicular. But in all the Harland & Wolff calculations, a length between perpendiculars of 850 ft. was used, and the length between each of station in the plans was based on dividing 850 ft. by 24.